Terminology :-

(1) Input
(2) Response / output
(3) Controller ← Proportional; PI → Proportional + Integrator; PID Proport^n + Integrator + differentiator.
(4) Error

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Mechanical Translational System

⇒ List of symbols used

(1) X = Displacement (m)
(2) V = velocity = $\frac{dx}{dt}$ ($m\,sec^{-1}$)

(3) a = acceleration = $\frac{dv}{dt}$ ms^{-2}.

(4) F = force applied = ma $kg\,ms^{-2}$ or N.

(5) f_m = opposing force applied by body of mass 'm'. in Newton.

6) f_k = elasticity of the spring in ~~Nehwon~~ Newton

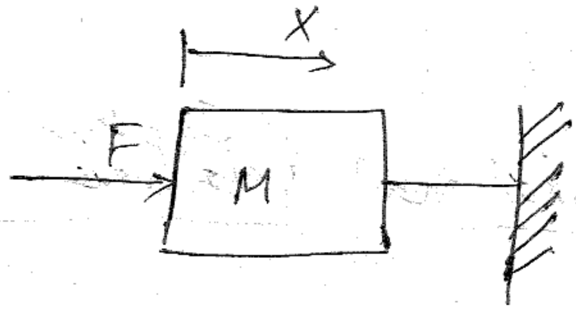
f_b = opposing force offered by dashpot element or frictional force. (Newton)

(9) K = Stifness of the spring
(N/m).

(10) B = ~~B~~ Viscous friction Coefficient
($N\,sec\,m^{-1}$)

(11) M = Mass of the body (kg)

(i)



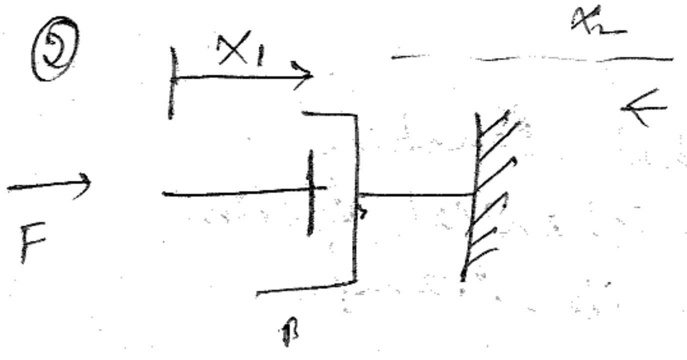
F = Applied force

F_m = ~~opt~~ opposing force offered by the body of mass 'm'.

$$f_m \propto \frac{d^2x}{dt^2}$$

$$f_m = m\frac{d^2x}{dt^2} \quad \text{— (1)}$$

(2)



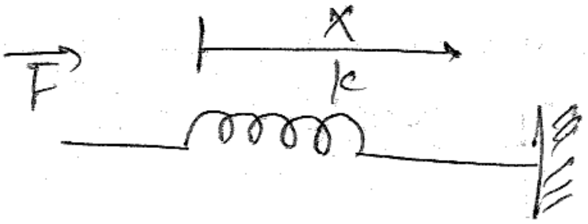
← Representⁿ of dash pot element

Case (i) Here, $f_b \propto \frac{dx}{dt} \Rightarrow \boxed{f_b = B\cdot\frac{dx}{dt}}$

Case ii)

$$f_b \propto \frac{d(x_1-x_2)}{dt}$$

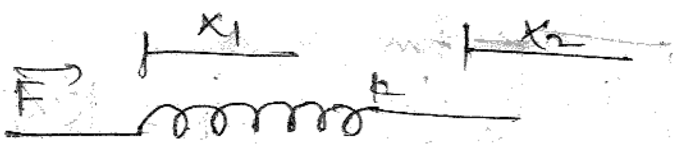
$$\boxed{f_b = B\,\frac{d(x_1-x_2)}{dt}}$$



$\cancel{f_k \propto \frac{dx}{dt}}$ $f_k \propto x$

$$\boxed{f_k = kx}$$

(3)



$$f_k \propto (x_1-x_2)$$

$$\boxed{f_k = k(x_1-x_2)}$$

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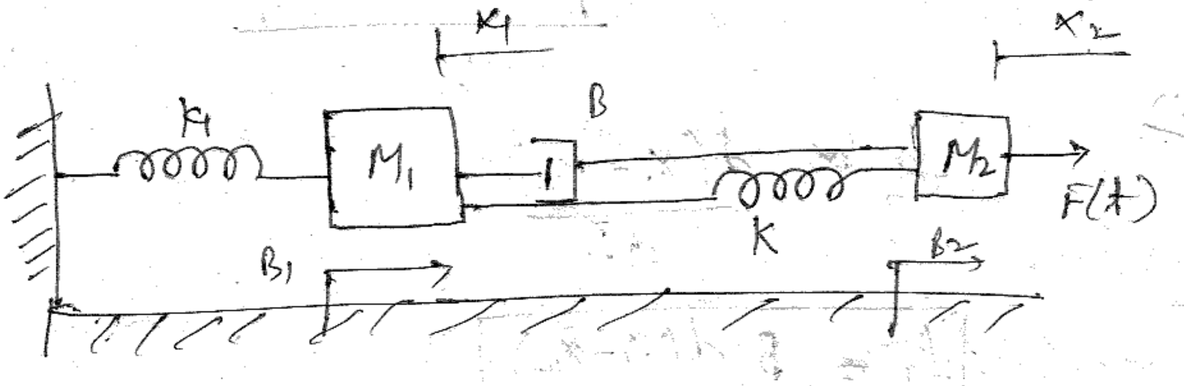
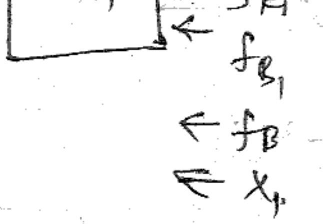
Q1 Write the differential Equations governing the mechanical systems and determine its transfer function



$$L\{F(t)\} = F(s)$$

$$L\left\{\frac{d^2x}{dt^2}\right\} = s^2 X(s)$$

$$L\left\{\frac{dx}{dt}\right\} = S \cdot X(s)$$



$$F_{m_1} = m_1 \frac{d^2x_1}{dt^2}$$

$$f_{K_1} = K x_1$$

$$f_{B_1} = B \frac{dx_1}{dt}$$

$$f_B = B_1 \frac{dx_1}{dt}$$

$$\Rightarrow m_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B\frac{d(x_1 - x)}{dt} + B_1 \frac{dx_1}{dt} + K(x_1 - x) = 0$$

Applying laplace transform in eqn ①, — ① eqn

we get,

$$\Rightarrow m_1 s^2 X_1(s) + K_1 X_1(s) + B s[X_1(s) - X(s)] + B_1 s X_1(s) + K[X_1(s) - X(s)]$$ — ② eqn

$$\Rightarrow m_1 s^2 X_1(s) + K_1 X_1(s) + Bs X_1(s) - Bs X(s) + B_1 s X_1(s) + K X_1(s) - K X(s) = 0$$

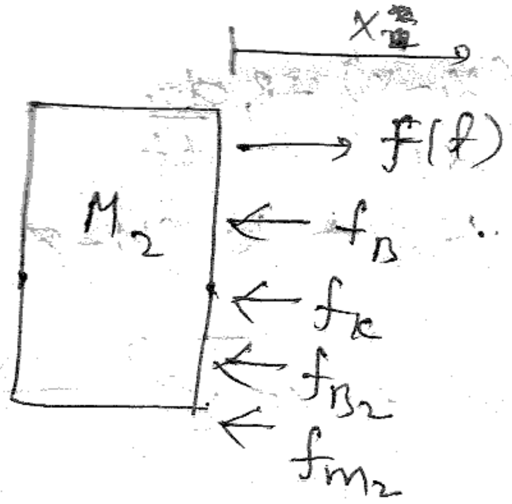
$$\Rightarrow m_1 s^2 X_1(s) + K_1 X_1(s) + Bs X_1(s) + B_1 s X_1(s) + K X_1(s)$$

$$\cancel{= BsX(s) + KX(s)}$$

$$= X(s)[Bs + K]$$

$$\Rightarrow X_1(s)\left[m_1 s^2 + (K_1 + K) + (B_1 + B)s\right] = X(s)[Bs + K]$$

$$\Rightarrow X_1(s) = \frac{X(s)[Bs + K]}{m_1 s^2 + (K_1 + K) + (B_1 + B)s}$$ — ③



$$f(t) = B_2\frac{dx}{dt} + M_2\frac{d^2x}{dt^2} + K(x-x_1) + B_2\frac{d(x-x_1)}{dt}$$

Taking laplace transform

$$F(s) = B\,s\,X(s) + m s^2 X(s) + K[X(s) - X_1(s)] + B_2[s\,X(s) - s\,X_1(s)]$$

$$= B s X(s) + m s^2 X(s) + K X(s) - K X_1(s) + B_2 s X(s) - B_2 s X_1(s)$$

$$\Rightarrow F(s) + K X_1(s) + B_2 s X_1(s) = B s X(s) + m s^2 X(s) + K X(s) + B_2 \cdot s X(s)$$

$\Rightarrow$

$$f(s) = B_2 s X(s) + m_2 s^2 X(s) + k[X(s) - X_1(s)] + B[sX(s) - sX_1(s)]$$

$$= B_2 s X(s) + m_2 s^2 X(s) + kX(s) - kX_1(s) + BsX(s) - BsX_1(s)$$

$$\Rightarrow F(s) - B_2 s X(s) - m_2 s^2 X(s) - kX(s) - BsX(s) = -kX_1(s) - BsX_1(s)$$

$$\Rightarrow B_2 s X(s) + m_2 s^2 X(s) + kX(s) - F(s) + BsX(s) = kX_1(s) + BsX_1(s)$$

$$\Rightarrow m_2 s^2 X(s) + B_2 s X(s) + kX(s) + BsX(s) - X_1(s)[k + Bs] = F(s)$$

$$\Rightarrow X(s)[m_2 s^2 + B_2 s + k + Bs] - X_1(s)[Bs + k] = F(s)$$

$$\Rightarrow X(s)[m_2 s^2 + (B+B_2)s + k] - X_1(s)[Bs + k] = F(s) \quad \text{—(4) eq}^n$$

Sub (3) in (4)

$$X(s)\left[m_2 s^2 + (B_1+B_2)\, s + K\right] - \frac{X_1(s)\,(Bs+k)^2}{m_1 s^2 + (B_1+B)s + (K_1+K)}$$

$$= F(s)$$

$$\Rightarrow X(s)\left\{m_2 s^2 + (B+B_2)s + k - \frac{(Bs+k)^2}{m_1 s^2 + (B_1+B)s + (k_1+k)}\right\}$$

$$= F(s)$$

$$\Rightarrow \boxed{\frac{X(s)}{F(s)} = m_2 s^2 + (B+B_2)s + K - \frac{(Bs+K)^2}{m_1 s^2 + (B_1+B)s + (k_1+k)}}$$

AZ

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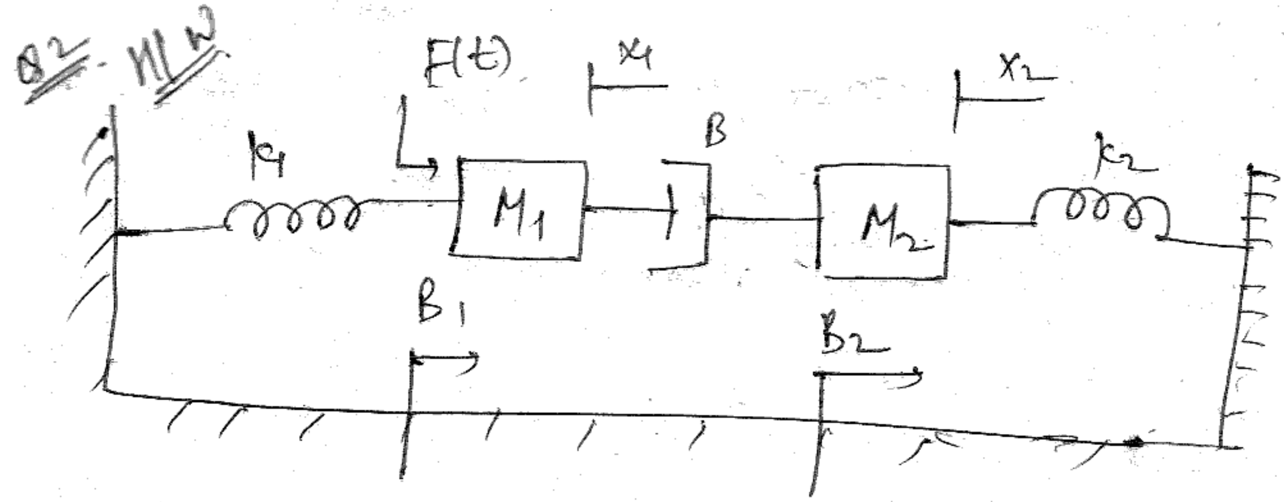
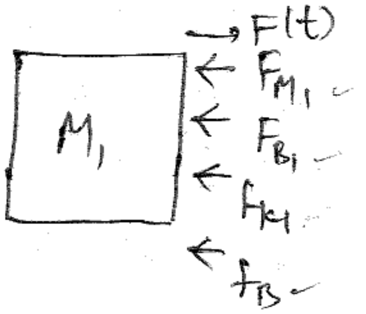
Q2. H/W.



$$\frac{X_1(s)}{F(s)} = ? \qquad \frac{X_2(s)}{F(s)} = ?$$



$B\frac{d(x_1-x_2)}{dt}$

F(t) =

$$M_1\frac{d^2x_1}{dt^2} + B\frac{dx_1}{dt} + B_1\frac{dx_1}{dt} + K_1 x_1 = 0$$

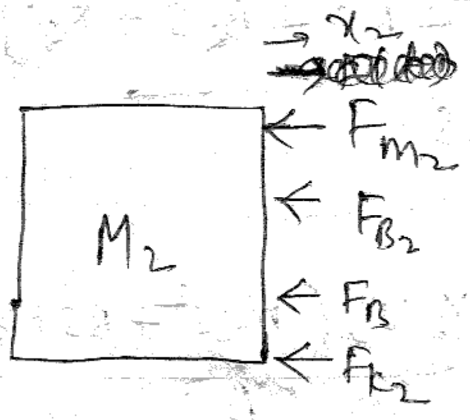
Apply Laplace transf.

$$M_1 s^2 X_1(s) + B s X_1(s) + B_1 s X_1(s) + K_1 X_1(s) = F(s)$$

$$\Rightarrow X_1(s)\left[M_1 s^2 + Bs + B_1 s + K_1\right] = F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{1}{m_1 s^2 + Bs + B_1 s + K_1}$$

$$\frac{X_1(s)}{F(s)} = \frac{1}{m_1 s^2 + (B_1 + B)s + K_1}$$



$$m_2 \frac{d^2 x_2}{dt^2} + B \frac{d(x_2 - x_1)}{dt} + B_2 \frac{dx_2}{dt} + K_2 (x_2) = 0$$

Taking laplace transform,

$$m_2 s^2 X_2(s) + B\left[s X_2(s) - s X_1(s)\right] + B_2 s X_2(s) + K_2 X_2(s) = 0$$

$$\Rightarrow m_2 s^2 X_2(s) + Bs X_2(s) - Bs X_1(s) + B_2 s X_2(s) + K_2 X_2(s) = 0$$

$$\Rightarrow m_2 s^2 X_2(s) + Bs X_2(s) + B_2 s X_2(s) + K_2 X_2(s) = Bs X_1(s)$$

$$\Rightarrow \frac{m_2 s^2 X_2(s) + B s X_2(s) + B_2 s X_2(s) + K_2 X_2}{B s} = X_1(s)$$

$$\Rightarrow X_1(s) = \frac{X_2(s)\left[m_2 s^2 + B s + B_2 s + K_2\right]}{B s}$$

$$\Rightarrow \frac{F(s)}{m_1 s^2 + (B_1 + B)s + K_1} = \frac{X_2(s)\left(m_2 s^2 + B s + B_2 s + K_2\right)}{B s}$$

$$\Rightarrow \frac{X_2(s)}{F(s)} = \frac{B s}{\left[m_1 s^2 + (B_1 + B)s + K_1\right]\left[m_2 s^2 + B s + B_2 s + K_2\right]}$$

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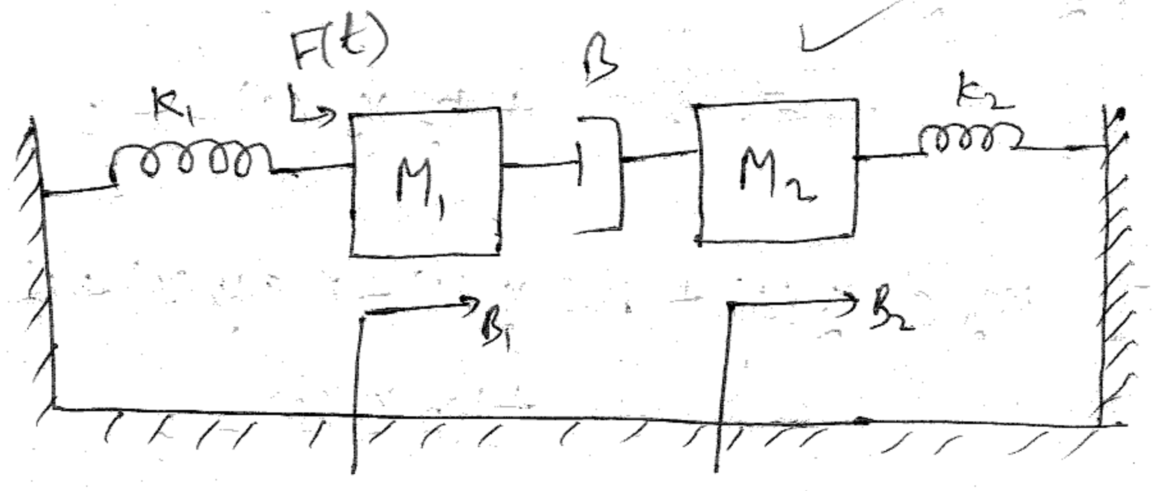
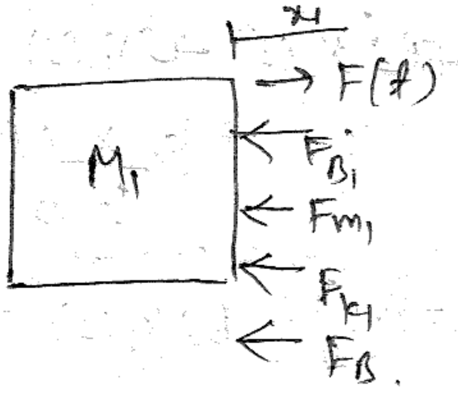
Q. 2



Find transfer funct^n $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$

$+K_2x_2$

Sol^n :- for Body of mass M_1,



$F_{m_1} = M_1 \frac{d^2x_1}{dt^2}$,

$F_{B_1} = B_1 \frac{dx_1}{dt}$, $\quad F_{k_1} = k_1 x_1$.

$F_B = \frac{d(x_1 - x_2)}{dt}$,

By Newtons second Law

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B\frac{d(x_1-x_2)}{dt} + k_1 x_1 = F(t)$$

$B_2s+k_2)$

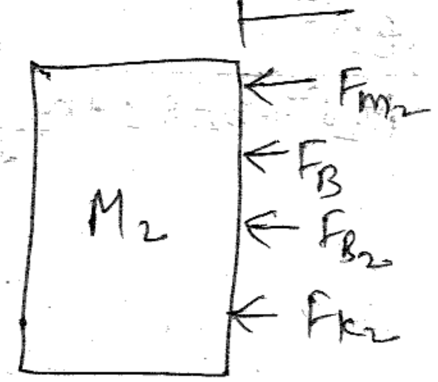
Applying Laplace Transf^n,

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + BsX_1(s) - BsX_2(s) + k_1 X_1(s) = F(s).$$

$Bs+B_2s$
$+k_2$

$$\Rightarrow X_1(s)[M_1 s^2 + B_1 s + Bs + k_1] - Bs X_2(s) = F(s) \quad -①.$$

For body of mass, M_2,



$F_{m_2} = M_2 \frac{d^2x_2}{dt^2}$,

$F_{B_2} = B_2 \frac{dx_2}{dt}$

$F_B = B\frac{d(x_2-x_1)}{dt}$,

$F_{k_2} = k_2 x_2$

$$\Rightarrow M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B\frac{d(x_2-x_1)}{dt} + k_2 x_2 = 0$$

$-②$

Taking Laplace transform,

$$\Rightarrow M_2 s^2 X_2(s) + B_2 s X_2(s) + Bs X_2(s) - Bs X_1(s) + K_2 X_2(s) = 0$$

$$\Rightarrow X_2(s)\left[M_2 s^2 + B_2 s + Bs + K_2\right] - Bs X_1(s) = 0$$

$$\Rightarrow X_2(s)\left[M_2 s^2 + (B_2 \cancel{s} + B)s + K_2\right] = Bs\, X_1(s)$$

$$\Rightarrow X_2(s) = \frac{Bs\, X_1(s)}{M_2 s^2 + (B_2 + B)s + K_2} \quad — (3)$$

Sub (3) in (1)

$$X_1(s)\left[M_1 s^2 + (B_1 + B)s + K_1\right] - \frac{(Bs)^2 X_1(s)}{M_2 s^2 + (B_2 + B)s + K_2} = F(s)$$

$$\Rightarrow \cancel{\frac{X_1(s)}{F(s)} =}$$

$$\Rightarrow X_1(s)\left[M_1 s^2 + (B_1 + B)s + K_1 - \frac{(Bs)^2}{M_2 s^2 + (B_2 + B)s + K_2}\right] = F(s)$$

$$\Rightarrow \cancel{\frac{X_1(s)}{F(s)} =}$$

$$X_1(s)\left[\frac{\left[M_1 s^2 + (B_1 + B)s + K_1\right]\left[M_2 s^2 + (B_2 + B)s + K_2\right] - (Bs)^2}{M_2 s^2 + (B_2 + B)s + K_2}\right] = F(s)$$

$$\Rightarrow \boxed{\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + (B_2 + B)s + k_2}{\left[M_1 s^2 + (B_1 + B)s + k_1\right]\left[M_2 s^2 + (B_2 + B)s + k_2\right] - (Bs)^2}}$$

from ③,

$$X_1(s) = \frac{X_2(s)\left[M_2 s^2 + (B + B_2)s + k_2\right]}{Bs} \quad \text{sub in ①}$$

$$\Rightarrow \frac{X_2(s)\left[M_2 s^2 + (B + B_2)s + k_2\right]}{Bs}\left[M_1 s^2 + (B_1 + B)s + k_1\right] - BsX_2(s) = F(s)$$

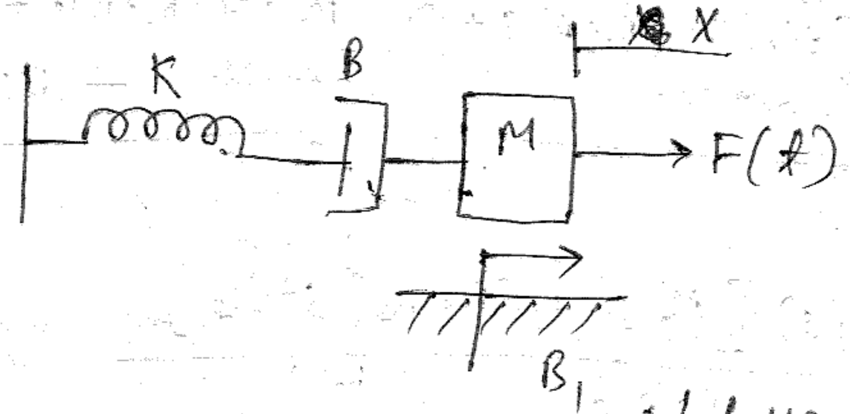
$$X_2(s)\left[\frac{\left[M_2 s^2 + (B + B_2)s + k_2\right]\left[M_1 s^2 + (B_1 + B)s + k_1\right] - (Bs)^2}{Bs}\right] = F(s)$$

$$\cancel{\frac{X_2(s)}{F(s)} = \frac{\left[M_2 s^2 + (B + B_2)s + k_2\right]\left[M_1 s^2 + (B_1 + B)s + k_1\right] - Bs^2}{Bs}}$$

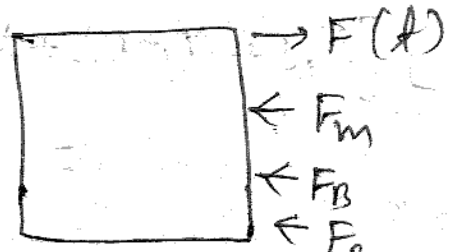
$$\boxed{\frac{X_2(s)}{F(s)} = \frac{Bs}{\left[M_2 s^2 + (B + B_2)s + k_2\right]\left[M_1 s^2 + (B_1 + B)s + k_1\right] - (Bs)^2}}$$

Ans

Q3



Let us assume the body of mass $M_1 = 0$ be connected bet: spring and dash pot



$$F(t) = M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + \cancel{B\frac{d(x-x_1)}{dt}}\ B\frac{d(x-x_1)}{dt}$$

Taking laplace transform

$$F(s) = Ms^2X(s) + B_1 s X(s) + sX(s) -$$

$$\Rightarrow F(s) = X(s)\left[Ms^2 + B_1 s + s\right]$$

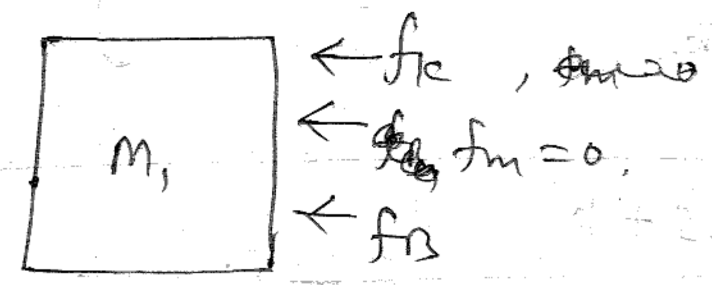
$$= X(s)\left[Ms^2 + (B_1+1)s\right]$$

$$F(s) = Ms^2X(s) + B_1 sX(s) + BsX(s) - BsX_1(s)$$

$$= X(s)\left[Ms^2 + B_1 s + Bs\right] - BsX_1(s)$$

$$F(s) = X(s)\left[Ms^2 + (B_1+B)s\right] - BsX_1(s) \quad \text{--- (1)}$$

for mass m_1



$$\Rightarrow 0 = B\frac{d(x_1 - x)}{dt} + Kx_1$$

Taking laplace transform,

$$BsX_1(s) - BsX(s) + KX_1(s) = 0$$

$$X_1(s)[Bs+K] = BsX(s)$$

$$X_1(s) = \frac{Bs\,X(s)}{Bs+K}$$

$$X_1(s) = \left(\frac{Bs}{Bs+K}\right)X(s) \quad — (2)$$

Sub in (1)

$$\Rightarrow X(s)\left[Ms^2+(B_1+B)s\right] - \frac{(Bs)^2\,X(s)}{Bs+K} = F(s)$$

$$\Rightarrow X(s)\left[Ms^2+(B_1+B)s - \frac{(Bs)^2}{Bs+K}\right] = F(s)$$

$$\cancel{\frac{X(s)}{F(s)} = Ms^2+(B_1+B)s - \frac{(Bs)^2}{Bs+K}}$$

$$X(s)\left[\frac{(Bs+k)\left[Ms^2+(B_1+B)s\right]-(Bs)^2}{Bs+k}\right]=F(s) \quad (7)$$

$$\Rightarrow \boxed{\frac{X(s)}{F(s)} = \frac{Bs+k}{(Bs+k)\left[Ms^2+(B_1+B)s\right]-(Bs)^2}}$$

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Mechanical Rotational Systems

List of Symbols :-

1) θ = Angular displacement in radians.

2) $\frac{d\theta}{dt}$ = Rate of change of angula displacem with respect to time

= Angular velocity (rad/sec)

3) $\frac{d^2\theta}{dt^2}$ = Angular Accelerat (rad/sec²)

4) T = Torque (Newton mtr) Nm

5) J = Moment of Inertia (Kg m²/rad)

6) B = Rotational friction coefficient (Nm or rad/sec)

7) K = Stiffness of spring (~~N/rad~~). Nm

T = torque.

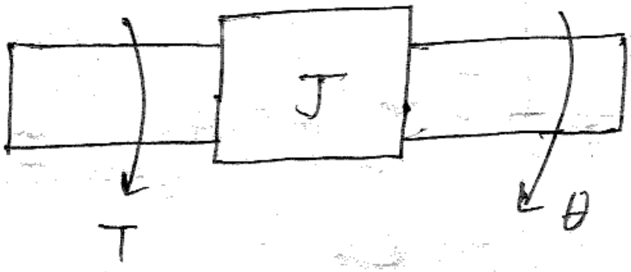
$F_m = T_j$

T_j = torque due inertia.

$\left[F_m = M \frac{d^2x}{dt^2} \right]$

The torque balance Eqn

①



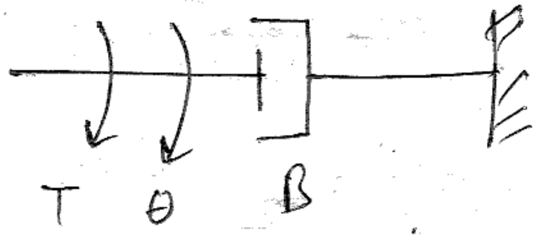
$$\boxed{T_j = J \cdot \frac{d^2\theta}{dt^2}}$$

T = Torque

T_j = Torque due to inertia.

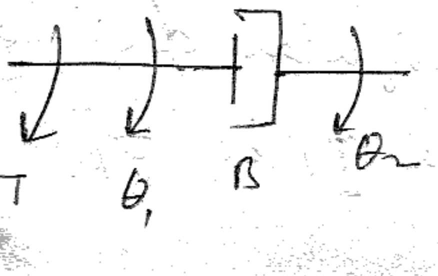
J = inertia

②



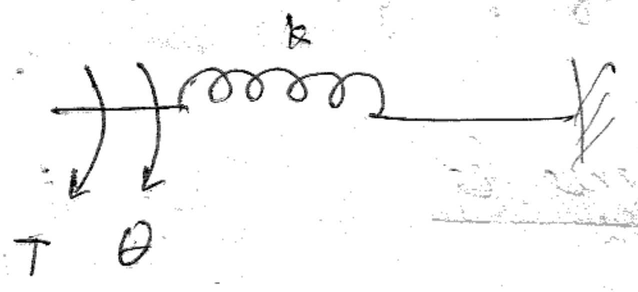
$$\boxed{T = T_B = B \cdot \frac{d\theta}{dt}}$$

③



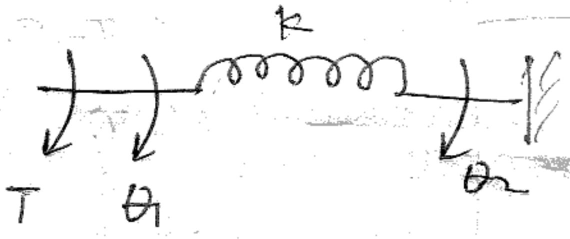
$$\boxed{T_B = B\frac{d(\theta_1 - \theta_2)}{dt}}$$

(5)



$$\boxed{T_k = K\theta}$$

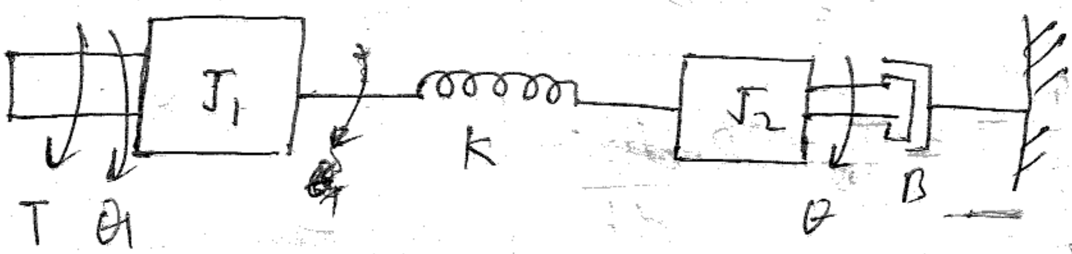
(6)



$$\boxed{T_k = k(\theta_1-\theta_2)}$$

where $\boxed{T = T_k}$.

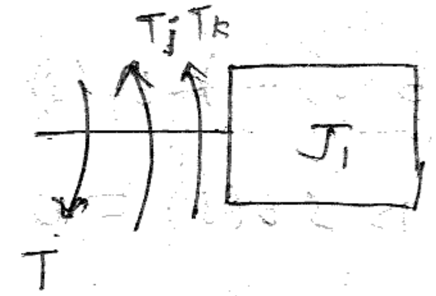
Q1



Find $\dfrac{\theta(s)}{T(s)}$

Let assume the angular displacements of body of inertia J_1 be θ_1

For J_1 :-



apply force $\rightarrow$ Clockwise.

$$\Rightarrow \quad T_j = J_j \frac{d^2\theta_1}{dt^2}, \quad T_k = k(\theta_1 - \theta)$$

$$T = T_j + T_k$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + k(\theta_1 - \theta)$$

$$T = J_j \frac{d^2\theta}{dt^2} + k\theta$$

Taking Laplace transform,

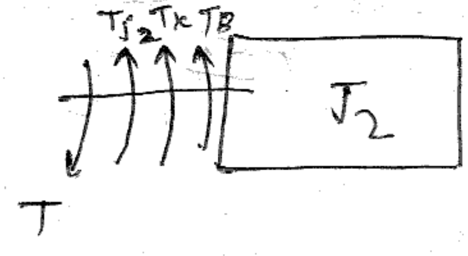
$$T(s) = J_1 s^2 \theta_1(s) + k\,\theta_1(s) - k\theta(s)$$

$$= J_1 s^2 \theta_1(s) - k\theta(s) + k\theta_1(s)$$

$$\cancel{= \theta(s)\left[J_1 s^2 + k\right] + k\theta_1(s)}$$

$$T(s) = \theta_1(s)\left[J_1 s^2 + k\right] - k\theta(s) \quad \text{--- (1)}$$

for 2nd body



$$T = T_{J_2} + T_k + T_b$$

$$= J_2 \frac{d^2\theta_2}{dt^2} + k(\theta - \theta_1) + B\frac{d(\theta)}{dt}$$

Taking laplace transform,

$$\cancel{T(s)} \Rightarrow J_2 s^2 \theta(s) + k\theta(s) - k\theta_1(s) + Bs\,\theta(s) = 0$$

$$\cancel{T(s)} \quad \theta(s)\left[J_2 s^2 + k + Bs\right] = k\theta_1(s)$$

$$\Rightarrow \boxed{\theta_1(s) = \frac{\theta(s)\left[J_2 s^2 + k + Bs\right]}{k}}$$

Sub ①

$$T(s) = \frac{\theta(s)\left[J_2 s^2 + k + Bs\right](J_1 s^2 + k)}{k} - k\theta(s)$$

$$= \theta(s)\left[\frac{(J_2 s^2 + k + Bs)(J_1 s^2 + k)}{k} - k\right]$$

$$\Rightarrow \boxed{\frac{\theta(s)}{T(s)} = \frac{k}{(J_2 s^2 + k + Bs)(J_1 s^2 + k) - k^2}}$$

Ans